SYMMETRIES AND ASYMPTOTES

RELATIONS AND AIDS IN SKETCHING GRAPHS

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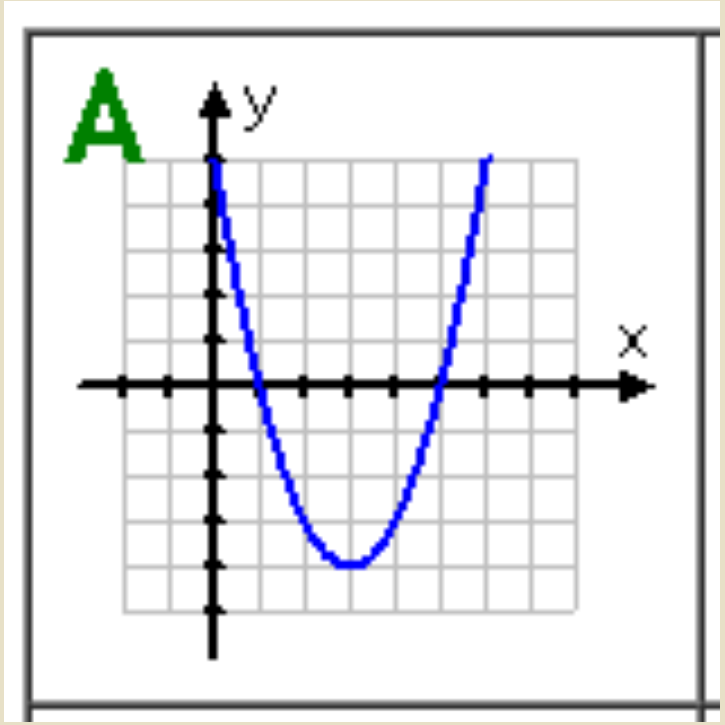
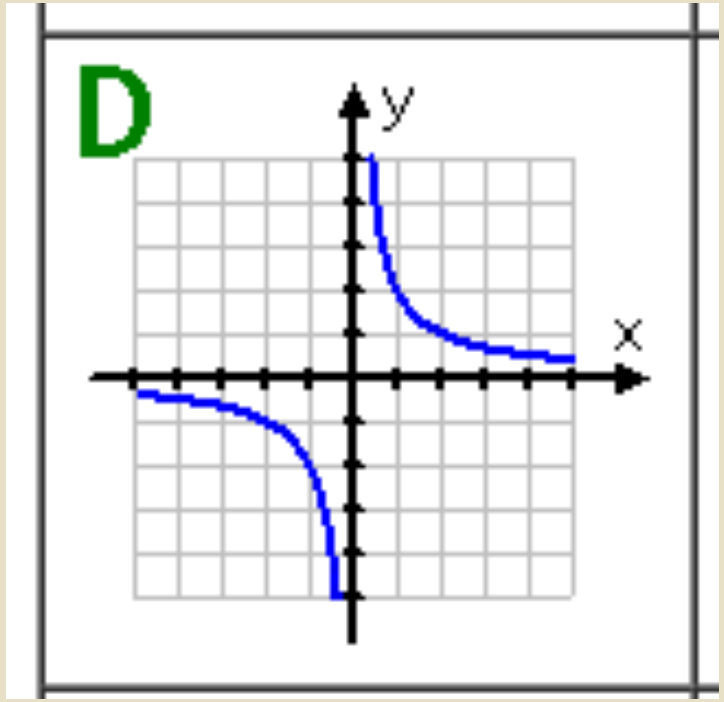
Even and Odd Functions

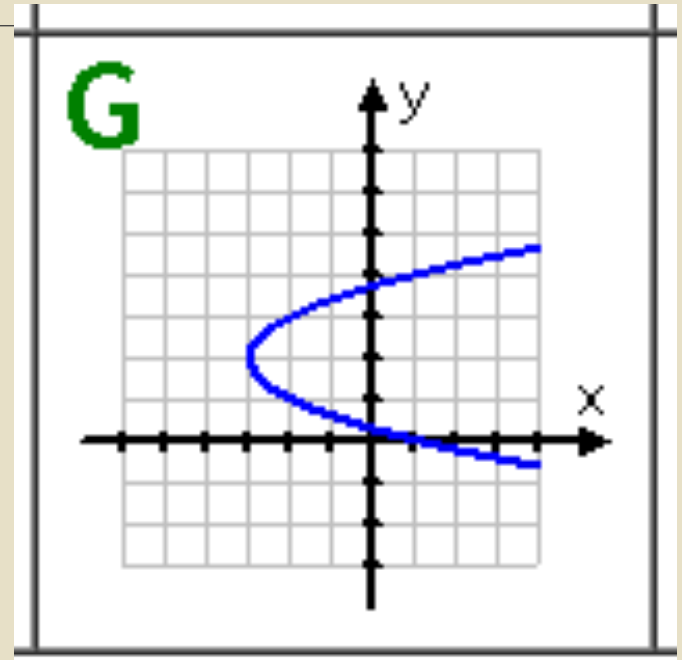
i. A function f is an **even function** if for every x in the domain of f, $f(-x) = f(x)$.

ii. A function f is an **odd function** if for every x in the domain of f, $f(-x) = -f(x)$.

Symmetry







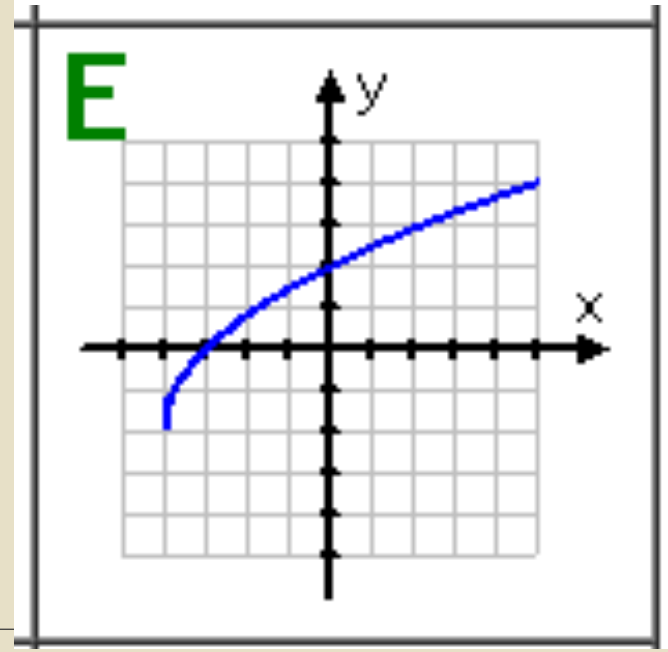


Image source: http://www.purplemath.com/modules/symmetry3.htm

Symmetry

i. A graph is said to be symmetric to the x-axis if whenever (a, b) is in the graph then so is $(a, -b)$

ii. A graph is said to be symmetric to the y-axis if whenever (a, b) is in the graph then so is $(-a, b)$

iii. A graph is said to be symmetric to the origin if whenever (a, b) is in the graph then so is $(-a, -b)$

Symmetry

i. Symmetric w.r.t. x-axis if we get an equivalent equation when all y's are replaced with –y's.

ii. Symmetric w.r.t. y-axis if we get an equivalent equation when all x's are replaced with –x's.

iii. Symmetric w.r.t. origin if we get an equivalent equation when all y's are replaced with –y's and all x's are replaced with –x's.

Symmetry

- An even function is symmetric with respect to the y-axis.
- An odd function is symmetric with respect to the origin.

Asymptote

- a line that continually approaches a given curve but does not meet it at any finite distance.

Asymptote

- Vertical Asymptote
- Horizontal Asymptote
- Slant (Oblique) Asymptote

Asymptote

- Vertical Asymptote
  - correspond to zeroes in the denominator
  - as x approaches to some constant value **C**, the function approaches to infinity

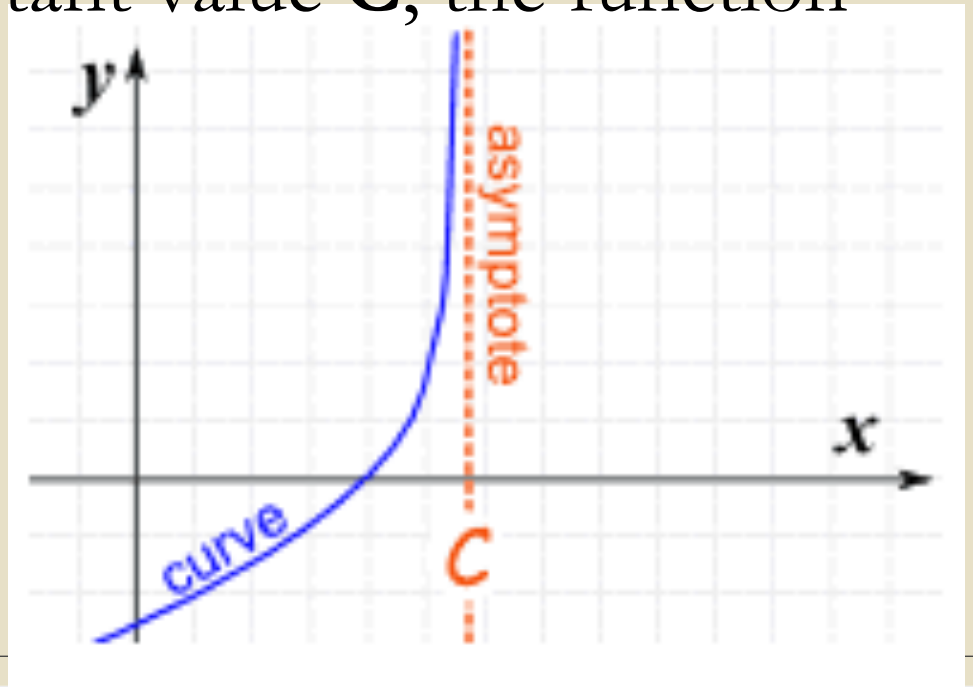


Image source: https://www.mathsisfun.com/algebra/asymptote.html

Asymptote

$$f(x) = \frac{x^2 + 2x - 3}{x^2 - 5x - 6}$$

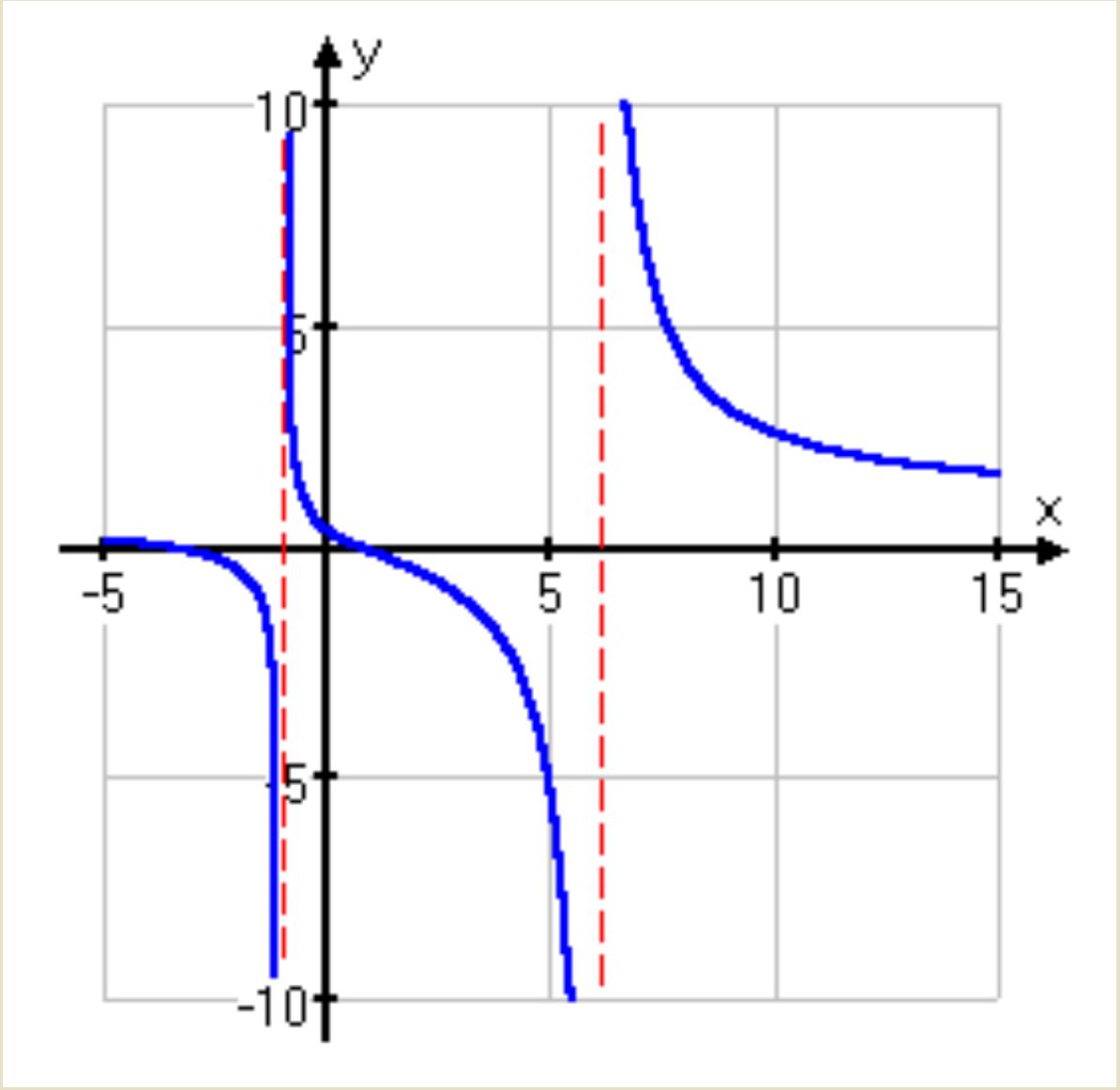


Image source: http://www.purplemath.com/modules/asymtote.htm

Asymptote

- For any rational function with n as the largest exponent of the numerator and m is the largest exponent of the denominator,

$$R(x) = \frac{ax^n + \cdots}{bx^m + \cdots}$$

If $n < m$ then the x-axis is the HA

If $n = m$ then the line $y = \frac{a}{b}$ is the HA

If $n > m$ then there is no HA

Asymptote

- Horizontal Asymptote
  - can be touched or crossed
  - as x approaches to infinity, the f
    certain constant value **b**

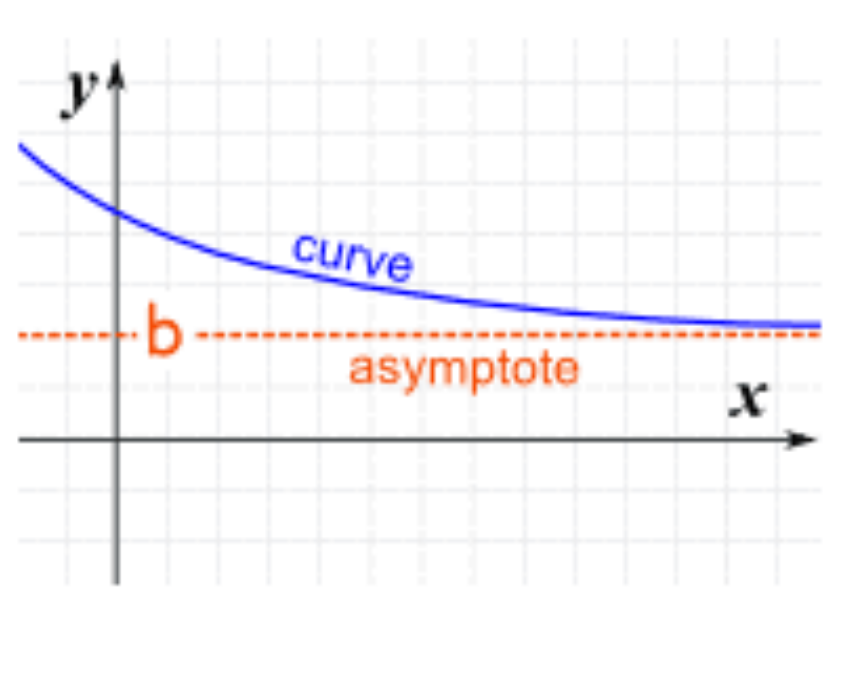


Image source: https://www.mathsisfun.com/algebra/asymptote.html

Asymptote

$$f(x) = \frac{x+2}{x^2+1}$$

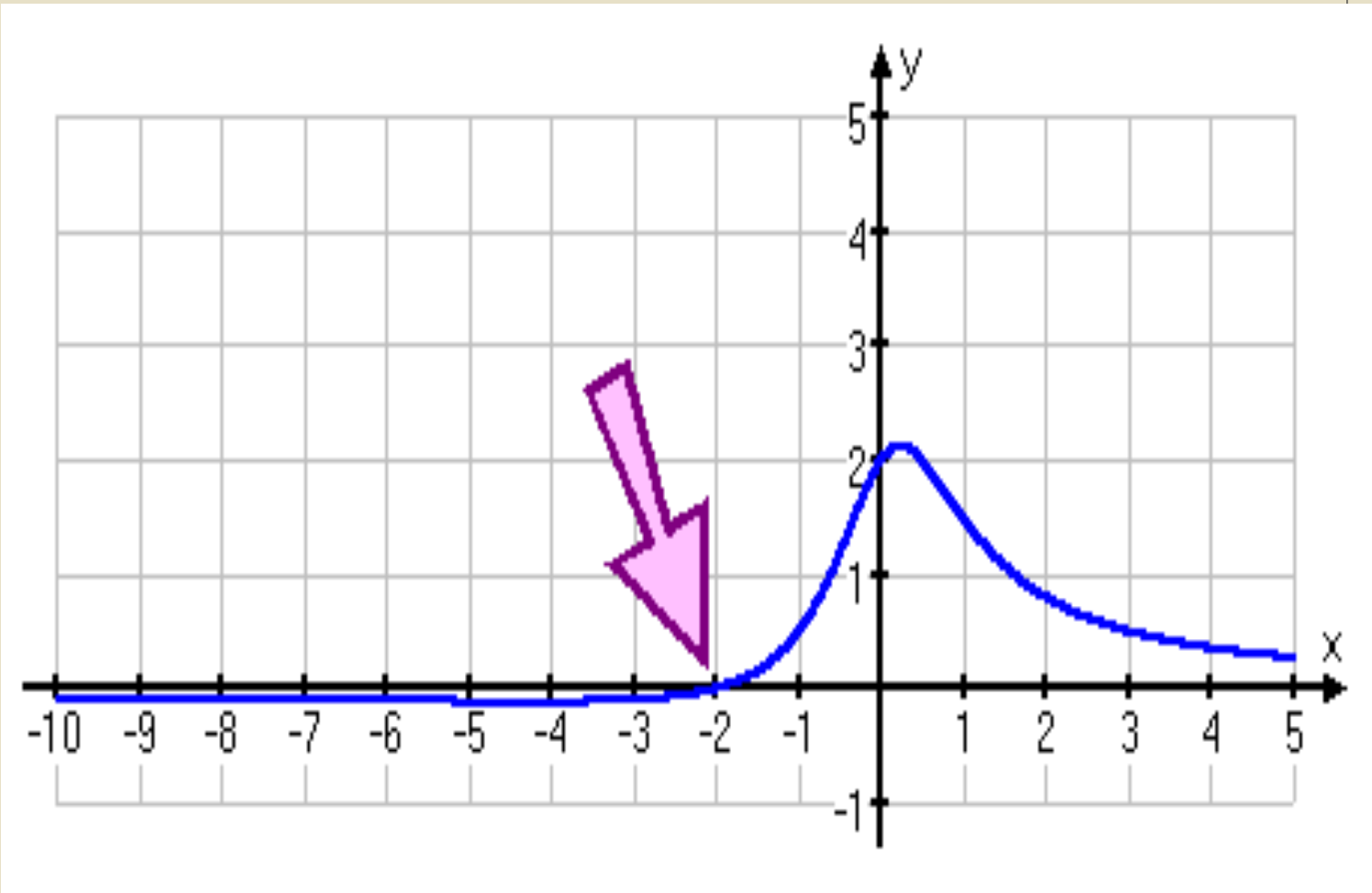
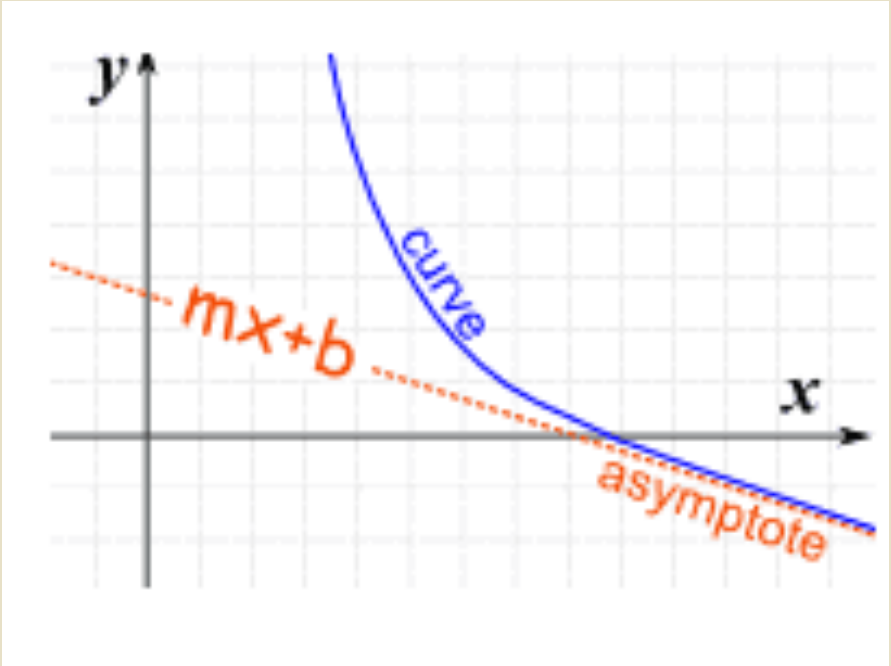


Image source: http://www.purplemath.com/modules/asymtote.htm

Asymptote

- Slant (Oblique) Asymptote
  - Happens on a rational function where the degree of the numerator is bigger than the denominator
  - as x approaches to infinity, the function goes towards a line y=mx+b



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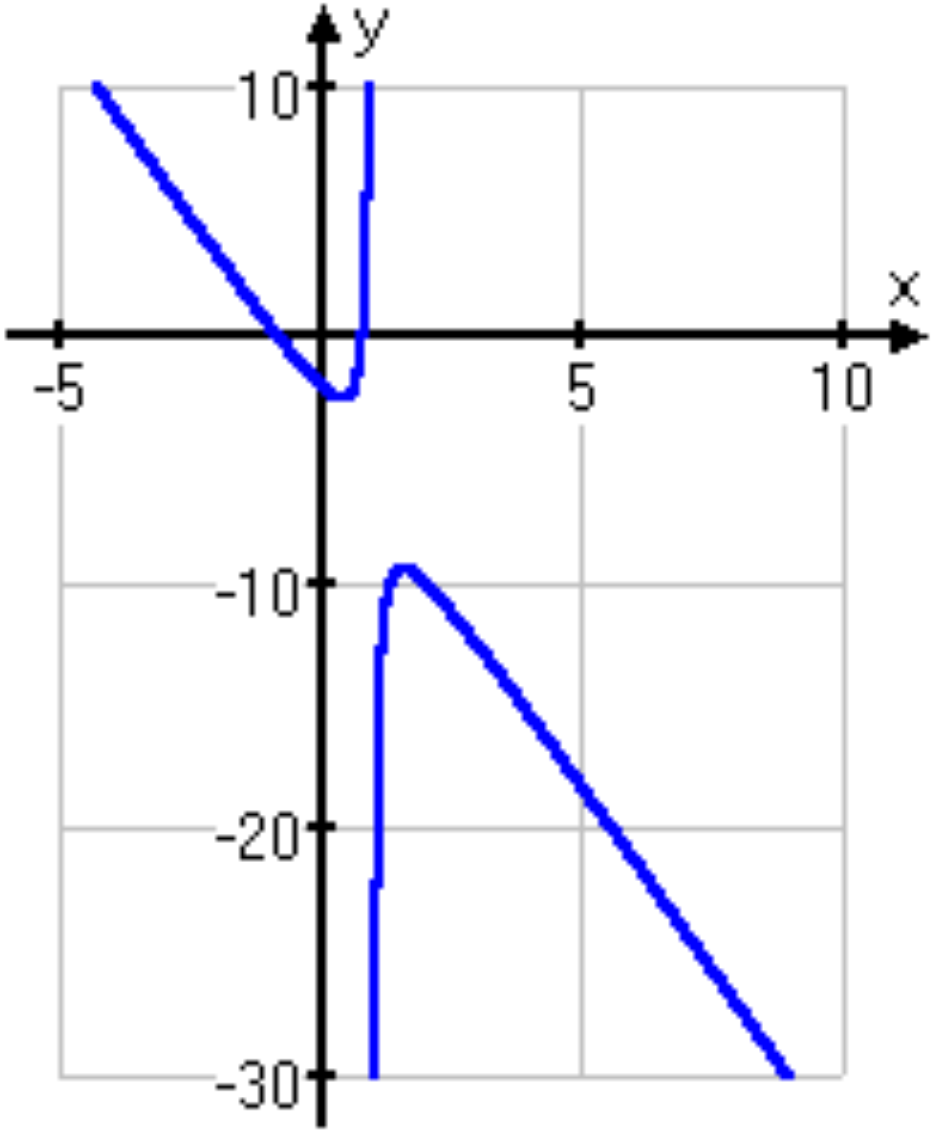
Asymptote

$$f(x) = \frac{-3x^2 + 2}{x - 1}$$



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Your Turn!

Find the domain of the functions. Determine also its symmetry and asymptotes (if any).

$$y = \frac{x^2 + 3x + 1}{4x^2 - 9}$$

$$y = \frac{x + 3}{x^2 + 9}$$

$$y = \frac{x^2 - x - 2}{x - 2}$$

References

- http://tutorial.math.lamar.edu/Classes/Alg/Symmetry.aspx#Graph_Sym_Ex1_a
- The Calculus 7 by Louis Leithold